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of which the first is Wilson's Theorem, while the left member of the second is the coefficient of $x^m y^{p-1-m}$ in $(x+y)^{p-1}$. The latter is congruent, modulo p, to

$$\frac{x^p+y^p}{x+y}=x^{p-1}-x^{p-2}y+\ldots\ldots+(-1)^m x^m y^{p-1-m}+\ldots\ldots+y^{p-1}.$$

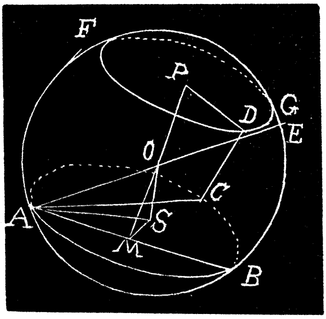
AVERAGE AND PROBABILITY.

130. Proposed by **LON C. WALKER. A. M.**, Graduate Student. Leland Stanford University, Cal.

Four points are taken at random on the surface of a given sphere; show that the average volume of a tetrahedron formed by the planes passing through the points taken three and three, is 1-35 of the volume of the given sphere.

I. Solution by the PROPOSER.

Choose A, B, C, D as the four random points; O the center of the given sphere with radius r: $ABFE$ a great circle through A, B; ABC a small circle through A, B, C, with center S; DGF a small circle through D parallel to ABC, with center P; M the middle point of AB.



Put $OP=x$, $AS=r_1$, $\angle AOB=\theta$, $\angle OMS=\phi$, $\angle CAB=\psi$, $\angle SAM=\psi_1$. Then we have

$$AM=r\sin\tfrac{1}{2}\theta=r_1\cos\psi_1,$$
$$SM=r\cos\tfrac{1}{2}\theta\cos\phi=r_1\sin\psi_1,$$
$$OS=r\cos\tfrac{1}{2}\theta\sin\phi,$$
$$AC=2r_1\cos(\psi-\psi_1),$$
$$PD=\sqrt{(r^2-x^2)},$$
$$r_1=r(\sin^2\tfrac{1}{2}\theta+\cos^2\tfrac{1}{4}\theta\cos^2\phi)^{\frac{1}{2}}, \text{ area } ABC=2rr_1\sin\tfrac{1}{2}\theta\sin\psi\cos(\psi-\psi_1),$$

volume of tetrahedron $D—ABC=\frac{1}{3}SP.\text{area}ABC=\frac{2}{3}rr_1(x+r\cos\frac{1}{2}\theta\sin\phi)\sin\frac{1}{2}\theta \times\sin\theta\cos(\psi-\psi_1)$.

Hence, we have for the required average volume, $V=\frac{4}{105}\pi r^3=\frac{1}{35}$ of the volume of the given sphere.

[For the integration, see solution in last issue, page 113, where the figure for Professor Zerr's solution was inserted for the one belonging to Professor Walker's solution. F.]

131. Proposed by **LON C. WALKER. A. M.**, Graduate Student, Leland Stanford Jr. University. Cal.

A sphere is described with its center within a given sphere, and its surface intersecting the surface of the given sphere. The average volume common to both spheres is 10/21 of the volume of the given sphere.

Solution by the PROPOSER.

Let M be the center of the given sphere, and N that of the random sphere. Put $BM=a$, $NB=x$, $MN=y$, $\angle MBN=\theta$, $\angle BMN=\phi$, $\angle BNM=\psi$. Then we have